Optimal Coupling Ratio Selection for Flexible Appendage Actuators

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Introduction

ARTICULATED, flexible spacecraft appendages often depend on rotary, gear-coupled actuators for their orientation. Appendages requiring precise pointing are most frequently driven by dc servomotors or stepper motors connected to backlashless gear trains with high coupling ratios. The process of selecting the coupling ratio has historically been dominated by efforts to minimize lost motion (backlash) and weight. These constraints have been met in low-static-friction, high-efficiency commercial gear designs, providing an opportunity to select a coupling ratio to improve the system dynamics. Methods for selecting gear ratios to minimize the power dissipated in moving a rigid, fixed based load have been developed[1] and are extended here to simultaneously minimize both the required actuator power and the vibrational coupling between a rigid-body spacecraft and a flexible load with one generally damped torsional degree of freedom.

System Model

Figure 1 diagrams a system representing a rigid-body spacecraft with inertia J_c, an appendage drive with motor inertia J_0, gear coupling ratio N, and a flexible appendage with inertia J_1 and torsional stiffness K. The spacecraft body is assumed to have negligibly small products of inertia; thus, it is possible to model the spacecraft motion with single-axis rotational dynamics. Linear, viscous damping is assumed for each rotational degree of freedom. Equations (1) and (2) give the constraint relationships for the ideal gear train with no lost motion and no static friction, shown in Fig. 1:

$$\theta_g = -1/N\theta_0 \tag{1}$$

$$T_g = -NT_0 \tag{2}$$

With these constraints and assumptions, the system equations of motion are

$$-T_m = J_c\ddot{\theta}_c + D_c\theta_c \tag{3}$$

$$T_m = J_0\ddot{\theta}_0 + D_0\dot{\theta}_0 \tag{4}$$

$$T_g = J_g\ddot{\theta}_g + K(\theta_g - \theta_1) \tag{5}$$

$$0 = J_1\ddot{\theta}_1 + D_1\dot{\theta}_1 + K(\theta_1 - \theta_g) \tag{6}$$

Combining Eqs. (1-6) and noting that, for a practical system, the gear inertia is negligible compared to the load inertia ($J_g \ll J_1$), the equations written in state variable form reduce to

$$M\ddot{x} + C\dot{x} + Kx = T_{\text{ext}} \tag{7}$$

Received August 12, 1988; revision received Feb. 14, 1989.

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Or, in matrix form,

$$\begin{bmatrix} -T_m \\ -NT_m \\ 0 \end{bmatrix} = \begin{bmatrix} J_c & 0 & 0 \\ 0 & -NJ_0 & 0 \\ 0 & 0 & J_1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_c \\ \ddot{\theta}_0 \\ \ddot{\theta}_1 \end{bmatrix} + \begin{bmatrix} D_c & 0 & 0 \\ 0 & -D_0 & 0 \\ 0 & 0 & D_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_c \\ \dot{\theta}_0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -K/N & -K \\ 0 & K/N & K \end{bmatrix} \begin{bmatrix} \theta_c \\ \theta_0 \\ \theta_1 \end{bmatrix} \tag{8}$$

System Transfer Functions

Taking the Laplace transform of the matrix equation [Eq. (8)] neglecting initial conditions, the transfer function from appendage angle (θ_1) to spacecraft attitude (θ_c) is derived:

$$\frac{\theta_c}{\theta_1} = \frac{(NJ_1J_0)(s^3 + s^2Q_2 + sQ_1 + Q_0)}{(KJ_c)(s + D_c/J_c)} \tag{9}$$

where

$$Q_2 = [D_0/J_0 + D_1/J_1]$$

$$Q_1 = [K(N^2J_0 + J_1)/N^2J_0J_1 + (D_0D_1/J_0J_1)]$$

$$Q_0 = [K(N^2D_0 + D_1)/N^2J_0J_1]$$

Similarly, the transfer function from motor torque T_m to θ_1 is

$$\frac{T_m}{\theta_1} = -\frac{s(s^3 + s^2Q_2 + sQ_1 + Q_0)}{(K/NJ_0J_1)} \tag{10}$$

The T_m is applied equally and with opposite sign to the spacecraft body and the actuator rotor, as shown in Fig. 1. The greatest interaction between the flexible appendage and the spacecraft occurs during the period of least efficient actuator operation. During this time, torque is applied by the actuator to accelerate the appendage, and most of the input power (all when the appendage is at rest) is dissipated as heat. Electromechanical actuators generate heat dissipation according to the following relationship[1]:

$$P_d = CT_m s^2 \tag{11}$$

The constant C is a function of the actuator properties and, for a correctly designed system, remains essentially constant during operation. For the electromechanical actuator, C is the quotient of winding resistance and torque constant squared.

Therefore, the gear ratio that requires the least motor torque to accomplish a given appendage motion [i.e., minimizes Eq.

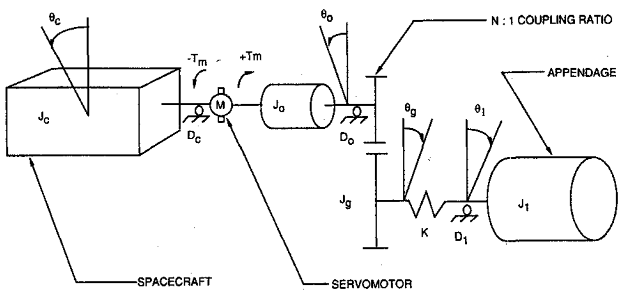

Fig. 1 Spacecraft, gear coupled actuator, and compliant appendage load.

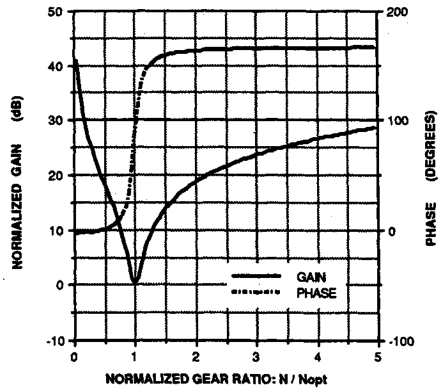

Fig. 2 Normalized magnitude and phase of Eq. (9) as a function of the normalized gear ratio.

Table 1 System parameters

$J_c = 1000.0$, kg-m^2	$D_c = 1.0$, N-m-s/rad
$J_0 = 5.0$, kg-m^2	$D_0 = 0.6$, N-m-s/rad
$J_1 = 20.0$, kg-m^2	$D_1 = 1.2$, N-m-s/rad
$K = 10.0$, N-m/rad	$C = 1.0$, W/N-m^2

(10)] will simultaneously minimize both the dynamic coupling between the spacecraft and the appendage [Eq. (9)] and the overall actuator power consumption [Eq. (11)].

Determination of the Optimum Coupling Ratio

The magnitudes of Eqs. (9) and (10) were expressed in the frequency domain ($s = j\omega$) and then differentiated with respect to N. Solution of these equations set equal to zero, neglecting viscous damping, resulted in the following condition for N at all frequencies:

$$N_{\text{opt}} = \sqrt{\frac{J_1}{J_0}} \tag{12}$$

Numerical Example

Table 1 gives representative parameter values for the system shown in Fig. 1, which was analyzed to show the result shown in Eq. (12) for a system with arbitrary viscous damping. Using these parameters, the maximum magnitude of Eqs. (9) and (10) as a function of N were calculated choosing driving frequencies five decades above and below the system natural frequency for incremental values of N. For every frequency, a distinct minimum occurred for $N = N_{\text{opt}}$ as given by Eq. (12). To show this effect, the gain (normalized to its magnitude at $N = N_{\text{opt}}$ and phase of Eq. (10) for a given frequency $s = j\omega$ are plotted as a function of the normalized coupling ratio in Fig. 2.

Results and Conclusion

Figure 2 shows the dramatic improvement in motor torque to appendage transmission for any given motion achieved by selecting the optimum gear coupling ratio as shown in Eq. (12). Using this method, typical lightly damped spacecraft appendages will exhibit improved controllability, a reduced level of required power during a maneuver, reduced dissipation, and reduced interaction with the attitude control dynamics.

Reference

[1]Truxal, J. G., *Control Engineers Handbook,* 1st ed., McGraw-Hill, New York, 1958, pp. 13.7–13.12 and 12.2–12.17.

Reduction of Missile Navigation Errors by Roll Programming

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Introduction

THE concept of rotating inertial instruments within a vehicle to allow averaging or cancellation of their errors has often been applied in the past. Stable platforms of the "carousel" type include an auxiliary turntable rotating continuously around the yaw axis, on which are mounted the instruments associated with the roll and pitch axes. (The yaw gyro is not mounted on the turntable, because its scale factor errors would eventually result in large yaw attitude errors.) And at least one strapdown system[1] rotates all of its instruments back and forth through ±360 deg. In both cases the rate of rotation is typically about 1 rpm.

In a missile employing strapdown instruments for navigation, a natural way of using the same principle is by changing the roll attitude of the missile itself, so that no hardware alterations are necessary. In this case, continuous rotation in one direction is not desirable because of the roll gyro scale factor (SF) error. Back and forth rotation will largely eliminate this effect, provided the SF error is a symmetrical one; for unsymmetrical SF errors it is desirable to minimize the total amount of roll motion. In addition, uplink communications may require upright (or inverted) roll attitudes for reasons of antenna polarization. The strategy considered here consists of simply introducing one or more periods of inverted flight into the trajectory, attained by relatively rapid rollovers so that the missile is nearly always either upright or inverted. In general, most of the achievable improvement can be realized by a simple schedule (such as a single inverted interval), preprogrammed at launch, based on the expected time of flight.

The technique can be applied to the reduction of chosen components of navigation error or, in the case of a terminal homing missile, to reduce seeker pointing errors (SPE) or heading errors at the expected time of target acquisition. We will concentrate here on reducing SPE. Further details can be found in Ref. 2.

The choice of an appropriate roll schedule is facilitated by deriving "influence functions" $h(t)$ that indicate the effect, on each component of SPE, of each reversible instrument error coefficient c_i:

$$\text{SPE}_i = \int_0^T c_i(t) h_i(t)\, dt \tag{1}$$

In Eq. (1), the coefficient c_i is constant in magnitude, but its sign changes when the missile is inverted. If we introduce the sign function

$$u(t) = \begin{cases} +1 & \text{(missile upright)} \\ -1 & \text{(missile inverted)} \end{cases} \tag{2}$$

we may express Eq. (1) as

$$\text{SPE}_i = c_i \int_0^T u(t) h_i(t)\, dt \tag{3}$$

where c_i is now constant.

Presented as Paper 88-4090 at the AIAA Guidance, Navigation, and Control Conference, Minneapolis, MN, Aug. 15–17, 1988; received Sept. 19, 1988; revision received Jan. 9, 1989.

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